RC Circuit Examples

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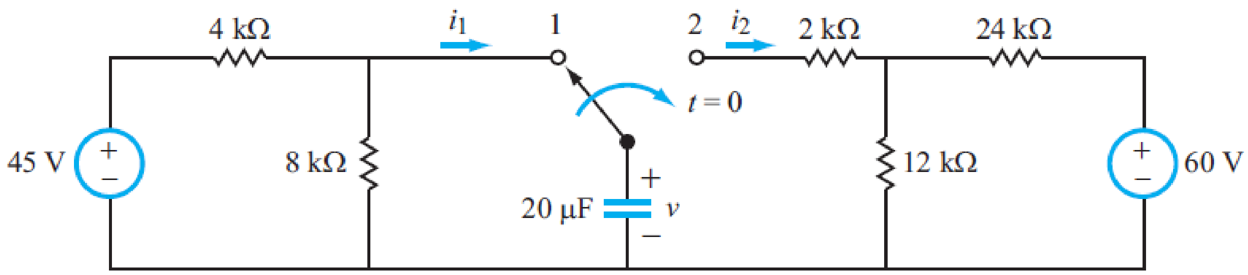
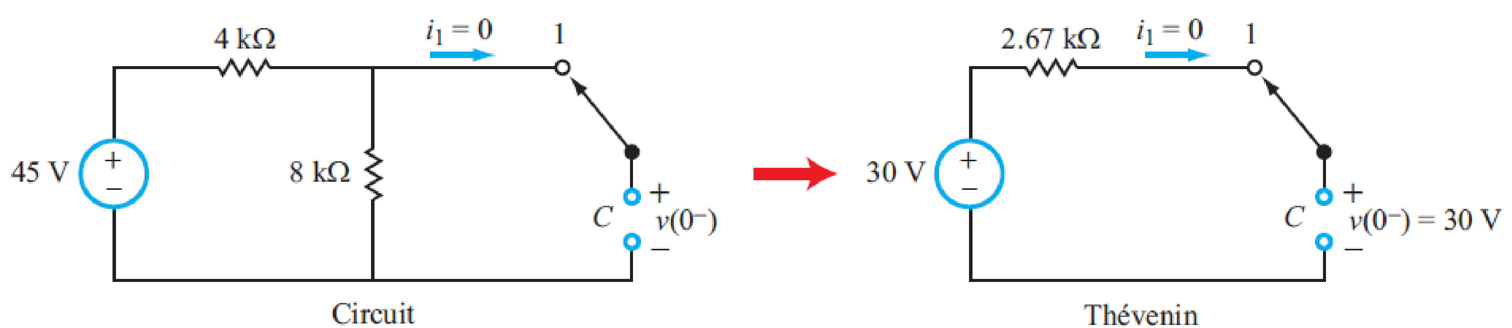
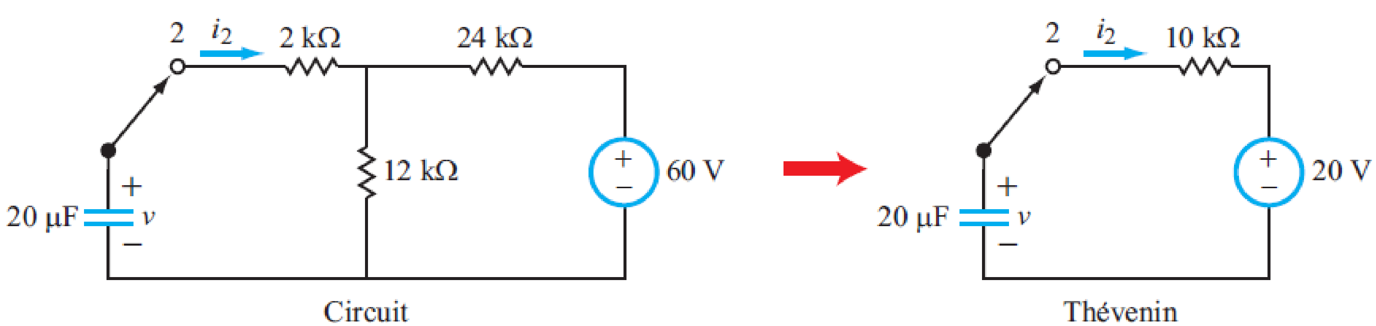
Example: Determine Capacitor Voltage



(a) Original circuit

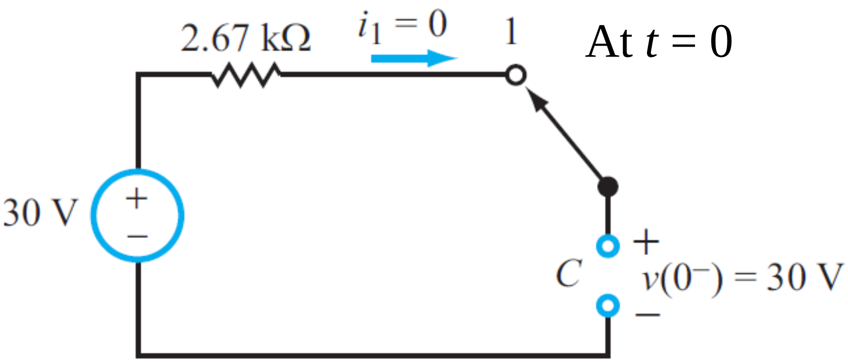


(b) At $t = 0^-$



(c) At $t \geq 0$

Solution



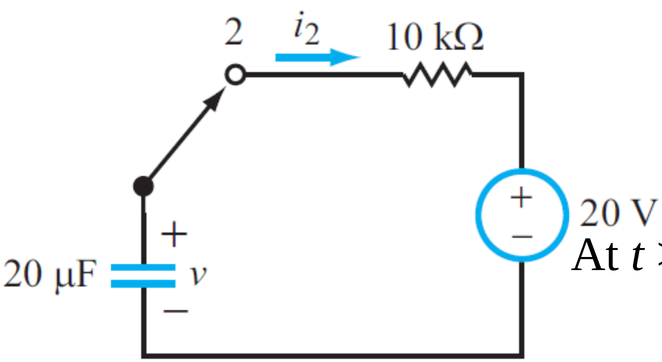
Thévenin
equivalent

At $t = 0$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$v(0) = v(0^-) = 30 \text{ V}$$

$$v(\infty) = \left(\frac{12\text{k}}{12\text{k} + 24\text{k}}\right) \times 60 = 20 \text{ V}$$



Thévenin
equivalent

At $t > 0$

$$R = R_{\text{Th}} = 2\ \text{k}\Omega + 12\ \text{k}\Omega \parallel 24\ \text{k}\Omega$$

$$= 2\ \text{k}\Omega + \frac{12\text{k} \times 24\text{k}}{12\text{k} + 24\text{k}} = 10\ \text{k}\Omega.$$

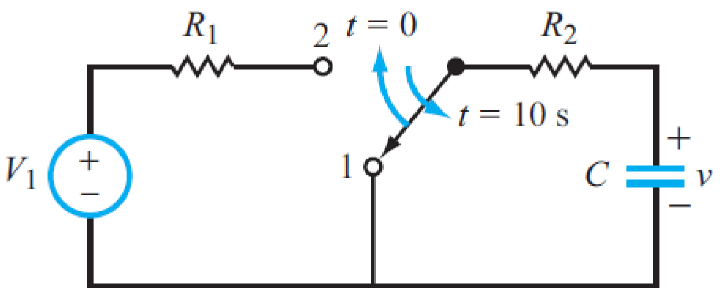
Hence,

$$\tau = RC = 10 \times 10^3 \times 20 \times 10^{-6} = 0.2 \text{ s}.$$

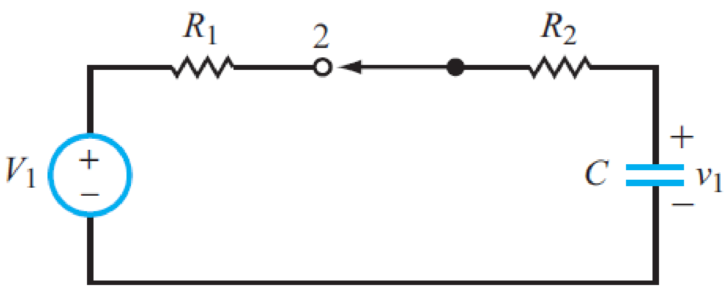
Substituting the values we obtained for $v(0)$, $v(\infty)$, and τ in Eq. (5.99) leads to

$$v(t) = (20 + 10e^{-5t}) \text{ V} \qquad (\text{for } t \geq 0).$$

Example: Charge/Discharge Action

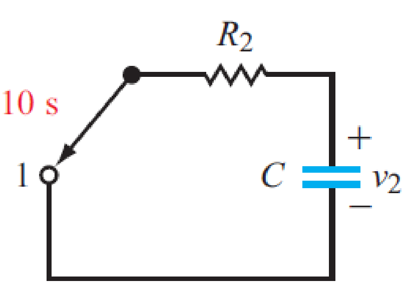


(a) Actual circuit



(b) Circuit during $0 \le t \le 10$ s

Circuit after $t = 10$ s



(c)

Given that the switch in Fig. 5-32 was moved to position 2 at $t = 0$ (after it had been in position 1 for a long time) and then returned to position 1 at $t = 10$ s, determine the voltage response $v(t)$ for $t \ge 0$ and evaluate it for $V_1 = 20$ V, $R_1 = 80$ kΩ, $R_2 = 20$ kΩ, and $C = 0.25$ mF.

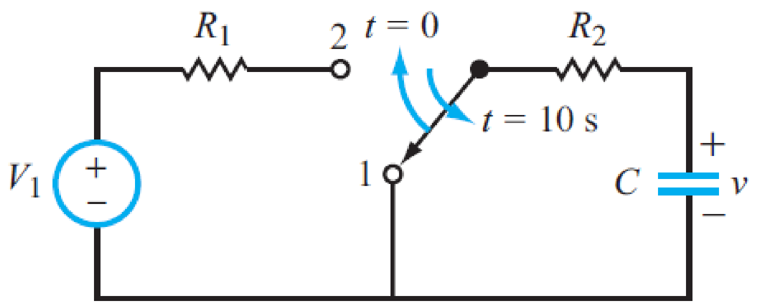
Time Segment 1: $0 \le t \le 10$ s

When the switch is in position 2 (Fig. 5-32(b)), the resistance of the circuit is $R = R_1 + R_2$. Hence, the time constant during this first time segment is

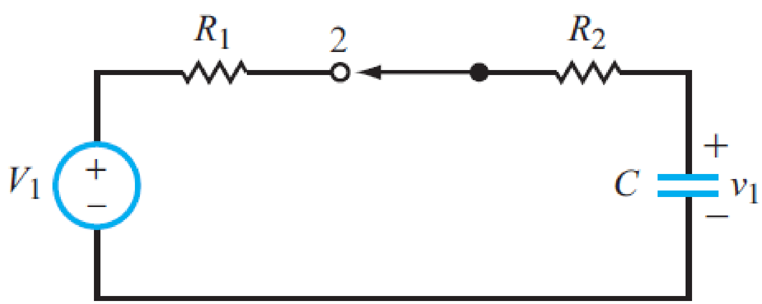
$$\tau_1 = (R_1 + R_2)C$$
$$= (80 + 20) \times 10^3 \times 0.25 \times 10^{-3} = 25 \text{ s}.$$

$$v_1(t) = v_1(\infty) + [v_1(0) - v_1(\infty)]e^{-t/\tau_1}$$
$$= 20(1 - e^{-0.04t}) \quad \text{V} \quad (\text{for } 0 \le t \le 10 \text{ s}).$$

Example (cont.)

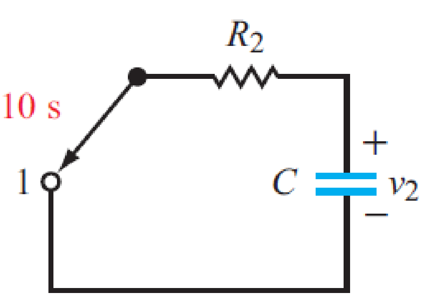


(a) Actual circuit



(b) Circuit during $0 \leq t \leq 10$ s

Circuit after $t = 10$ s



(c)

Time Segment 2: $t \geq 10$ s

Voltage $v_2(t)$, corresponding to the second time segment (Fig. 5-32(c)), is given by Eq. (5.98) with a new time constant τ_2 as

$$v_2(t) = v_2(\infty) + [v_2(10) - v_2(\infty)]e^{-(t-10)/\tau_2}.$$

The new time constant is associated with the capacitor circuit remaining after returning the switch to position 1,

$$\tau_2 = R_2 C$$
$$= 20 \times 10^3 \times 0.25 \times 10^{-3} = 5 \text{ s}.$$

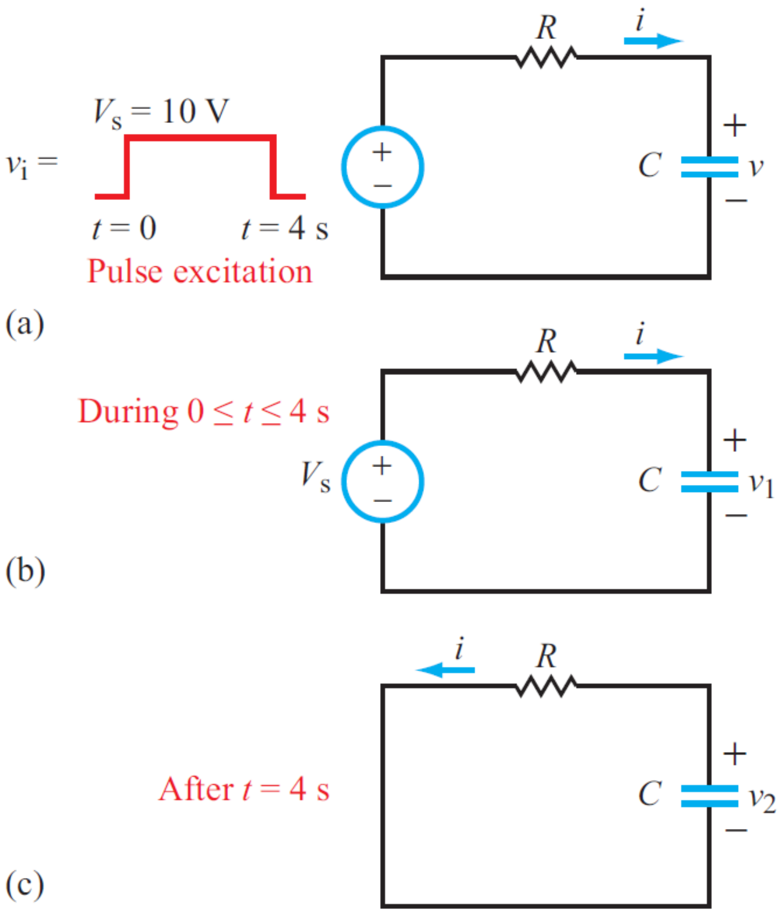
The initial voltage $v_2(10)$ is equal to the capacitor voltage v_1 at the end of time segment 1, namely

$$v_2(10) = v_1(10) = 20(1 - e^{-0.04 \times 10})$$
$$= 6.59 \text{ V}.$$

With no voltage source present in the R_2C circuit, the charged capacitor will dissipate its energy into R_2, exhibiting a ***natural response*** with a final voltage of $v_2(\infty) = 0$. Consequently,

$$v_2(t) = v_2(10)\ e^{-(t-10)/\tau_2}$$
$$= 6.59 e^{-0.2(t-10)} \text{ V} \quad (\text{for } t \geq 10 \text{ s}).$$

Example: Rectangular Pulse



$$v_i(t) = V_s\, u(t) - V_s\, u(t-4).$$

Since the circuit is linear, we can apply the superposition theorem to determine the capacitor response $v(t)$. Thus,

$$v(t) = v_1(t) + v_2(t),$$

$$v_1(t) = v_1(\infty) + [v_1(0) - v_1(\infty)]e^{-t/\tau}$$
$$= V_s(1 - e^{-t/\tau}) \qquad (\text{for } t \ge 0).$$

For $V_s = 10$ V and $\tau = RC = 25 \times 10^3 \times 0.2 \times 10^{-3} = 5$ s,

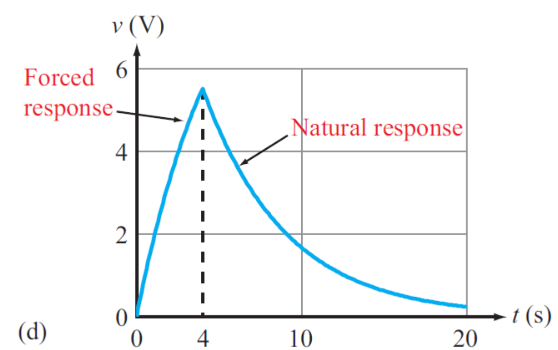
$$v_1(t) = 10(1 - e^{-0.2t}) \text{ V} \qquad (\text{for } t \ge 0).$$

The second step function has an amplitude of $-V_s$ and is delayed in time by 4 s. Upon reversing the polarity of V_s and replacing t with $(t-4)$, we have

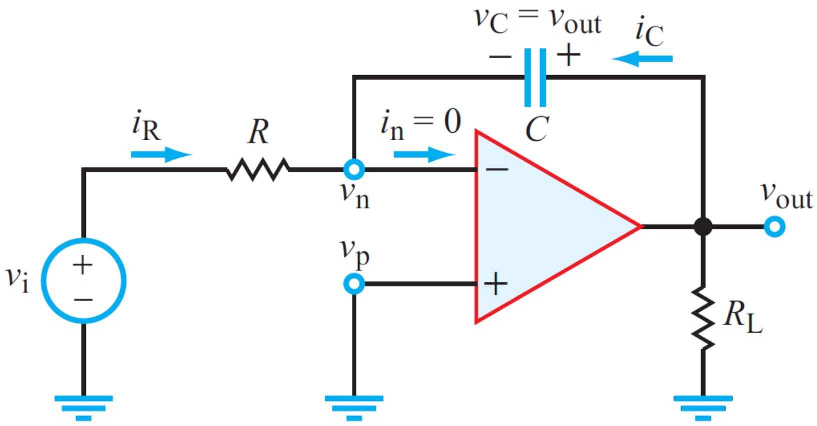
$$v_2(t) = -10[1 - e^{-0.2(t-4)}] \text{ V} \qquad (\text{for } t \ge 4 \text{ s}).$$

The total response for $t \ge 0$ therefore is given by

$$v(t) = v_1(t) + v_2(t)$$
$$= 10[1 - e^{-0.2t}] - 10[1 - e^{-0.2(t-4)}]\, u(t-4) \text{ V}$$



RC Op-Amp Circuits: Ideal Integrator



The output voltage v_{out} of such an integrator circuit is directly proportional to the time integral of the input signal v_i.

$$i_R = \frac{v_i}{R}. \tag{5.123}$$

Given that $v_n = 0$, the voltage v_C across C is simply v_{out}, and the current flowing through it is

$$i_C = C\,\frac{dv_{out}}{dt}. \tag{5.124}$$

At node v_n,

$$i_R + i_C - i_n = 0. \tag{5.125}$$

In view of the second op-amp constraint, namely $i_n = i_p = 0$, it follows that

$$i_C = -i_R \tag{5.126}$$

or

$$\frac{dv_{out}}{dt} = -\frac{1}{RC}\,v_i. \tag{5.127}$$

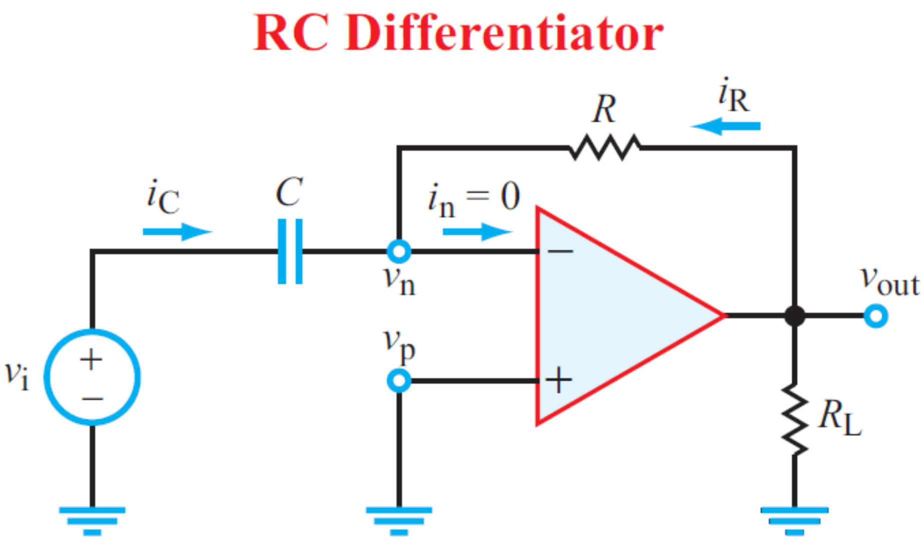
Upon integrating both sides of Eq. (5.127) from a reference time t_0 to time t, we have

$$\int_{t_0}^{t} \left(\frac{dv_{out}}{dt}\right) dt = -\frac{1}{RC}\int_{t_0}^{t} v_i\,dt, \tag{5.128}$$

which leads to

$$v_{out}(t) = -\frac{1}{RC}\int_{t_0}^{t} v_i\,dt + v_{out}(t_0). \tag{5.129}$$

RC Op-Amp Circuits: Ideal Differentiator



$$i_C = C\,\frac{dv_i}{dt},$$

$$i_R = \frac{v_{out}}{R},$$

$$i_C = -i_R.$$

$$v_{out} = -RC\,\frac{dv_i}{dt},$$